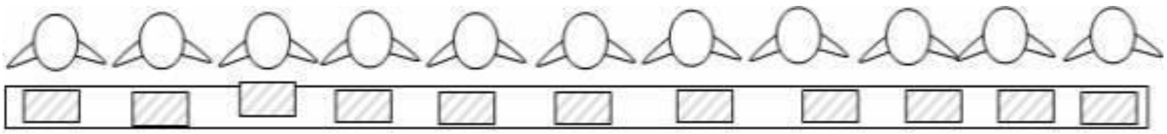
PROOF

As foot-noted in the book (Measuring Operators' Performance), here is the proof of the formula that gives the overall availability of line comprised with n identical stations and no intermediate buffer.

All stations are identical so are their individual availability (called d).



In general, the availability of a station can be characterized by two numbers: the Mean Time To Fail (MTTF) and the Mean Time to Repair (MTTR). The first number shows how often a problem (e.g., a breakdown) occurs on the station. The second number gives the average time it takes to fix a problem when it occurs. The reverse of MTTF and MTTR are respectively called failure rate and repair rate.

There are two useful ways to express the relationship between the availability d, the Mean Time To Fail (MTTF) and the Mean Time To Repair (MTTR).

Equation 1:

$$d = MTTF / (MTTF + MTTR) \quad (1)$$

Equation 2, is a simple rewriting of Equation 1:

$$MTTR / MTTF = (1/d - 1) \quad (2)$$

To compute the overall availability of the line, d_Line, we need to know MTTF_Line and MTTR_Line of the line. We work on the general case of a line comprise of n stations.

Computing the Mean Time To Fail of the line: MTTF_Line

Since there is no intermediate buffer, the whole line will stop any time there is a failure which occurs at one of the n stations. Therefore the failure rate of the whole line is given by the sum of failure rate of each line. This means:

$$1/ MTTF\_Line = n \times 1/ MTTF \quad (3)$$

or

$$MTTF\_Line = MTTF/n \quad (4)$$

This equation simply means that frequency of failures in the line will be multiplied by the number of the machines. In practice, it translates into the fact that the more machines you have the more problems or breakdowns you have, which is simply common sense.

Computing the Mean Time To Repair of the line: MTTR_Line

The mean time to repair of the whole line is the weighted average of the mean time to repair of all machines. Since all machines are identical, they are all failing at the same failure rate of 1/MTTF. This means:

MTTR_Line = MTTR (5)

This equation means that the overall Mean Time To Repair of the line will not depend on the number of machines. In practice, this is mostly driven by the repairing squad or maintenance team performance and organization.

Computing the availability of the whole line

We may now compute d_Line, the availability of the whole line as a function of the availability d of identical stations.

From Equation (1), we may compute d_line as follows:

d_Line = MTTF_Line / (MTTF_line + MTTR_Line) (6)

By rewriting a little bit we may obtain the following:

d_Line = 1/ (1 + MTTR_Line / MTTF_Line) (7)

Now using Equations (2), (4) and (5), we may also rewrite d_Line as follows:

d_Line = 1/(1+n x(1/d-1)) (8)

This equation means that at the end of the day, the overall efficiency of your line (d_line may also be call efficiency) will solely depend on the efficiency of each machine. You may get to the same result by two ways: 1/ making sure that your frequent breakdowns, if they cannot completely eliminated, are repaired quickly or 2/ making sure that the longest breakdowns are the least frequent.

Please note that the product of availabilities of the machines is a simple and very good approximate of the overall line efficiency (Equation 8) when machines' availabilities are close to 1 or/and when there are fewer machines in the line.